

CLAIMS

What is claimed is:

1. A computer-implemented method for approximating a function of an input argument, comprising:
 - selecting one of a plurality of breakpoints, such that a reduced argument for the function is less than a predetermined value; and
 - evaluating an approximate function of the reduced argument, including accessing a look-up table based on the selected breakpoint to obtain a value of a term in the approximate function,
 - wherein the look-up table has at least one breakpoint for which the reduced argument can be computed without roundoff error when the input argument is close to a root of the function.
2. The method of claim 1 wherein the function is $\log_b(X)$.
3. The method of claim 2 further comprising:
 - representing X in the floating point form $Y \cdot G^k$ where Y is greater than or equal to 1, and wherein the reduced argument is $Z = C \cdot (Y \cdot B_j - 1)$ where C is a function of $\log_b(e)$, and evaluating the approximate function includes determining $\log_b(1/B_j)$ using the look-up table and determining $\log_b(X)$ as an arithmetic combination of at least $k \cdot \log_b(2)$, $\log_b(1/B_j)$, and $\log_b(1 + Z/C)$.
4. The method of claim 3 wherein $Y \leq 2$ and the look-up table is modified such that $B_0 = 1$ and $B_N = 1/2$.
5. The method of claim 3 wherein $\log_b(1/B_j)$ is given by the look-up table as at least two lower precision values $T_{j,hi}$ and $T_{j,lo}$ whose sum equals $\log_b(1/B_j)$, $\log_b(2)$ is given by at least two lower precision values L_{hi} and L_{lo} whose sum equals $\log_b(2)$, and Z is given by at least two lower precision values Z_{hi} and Z_{lo} whose sum equals Z .
6. The method of claim 5 wherein $\log_b(X)$ is approximated by $A_1 + A_2 + Z_{lo}$, where A_1 is $k \cdot L_{hi} + T_{j,hi} + Z_{hi}$, A_2 is $k \cdot L_{lo} + T_{j,lo} + P$ and P is $\log_b(1 + Z/C) - Z$.

7. The method of claim 6 wherein if $k*N+j=0$ for the breakpoint, then $\log_b(X)$ is approximated by $(A_1+Z_{j0})+A_2$.
8. The method of claim 7 wherein $\log_b(X)$ is otherwise given by $A_1 + (A_2+Z_{j0})$.
9. The method of claim 3 wherein the predetermined value is proportional to $1/(2*N)$.
10. The method of claim 9 wherein $k*L_{hi}+T_{jhi}$ can be represented without roundoff error for all valid values of k,j .
11. The method of claim 10 wherein $T_{0,hi}=T_{0,lo}=0$ and $T_{N,hi}=L_{hi}$, $T_{N,lo}=L_{lo}$.
12. An article of manufacture, comprising:
a machine readable medium having instructions stored therein that can be executed by a processor to approximate a function of an input argument by selecting one of a plurality of breakpoints, such that a reduced argument for the function is less than a predetermined value, and evaluating an approximate function of the reduced argument including accessing a look-up table based on the selected breakpoint to obtain a value of a term in the approximate function, wherein the look-up table has at least one breakpoint for which the reduced argument can be computed without roundoff error when the input argument is close to a root of the function.
13. The article of manufacture of claim 12 wherein the function is $\log_b(X)$.
14. The article of manufacture of claim 13 wherein the medium has further instructions for representing X in the floating point form $Y*G^k$ where Y is greater than or equal to 1, and wherein the reduced argument is $Z=C*(Y*B_j-1)$ where C is a function of $\log_b(e)$, and evaluating the approximate function includes determining $\log_b(1/B_j)$ using the look-up table and determining $\log_b(X)$ as an arithmetic combination of at least $k*\log_b(2)$, $\log_b(1/B_j)$, and $\log_b(1+Z/C)$.
15. The article of manufacture of claim 14 wherein $Y \leq 2$ and the look-up table is modified such that $B_0=1$ and $B_N=1/2$.

16. The article of manufacture of claim 13 wherein $\log_b(1/B_j)$ is given by the look-up table as at least two lower precision values $T_{j,hi}$ and $T_{j,lo}$ whose sum equals $\log_b(1/B_j)$, $\log_b(2)$ is given by at least two lower precision values L_{hi} and L_{lo} whose sum equals $\log_b(2)$, and Z is given by at least two lower precision values Z_{hi} and Z_{lo} whose sum equals Z .

17. The article of manufacture of claim 16 wherein $\log_b(X)$ is approximated by $A_1 + A_2 + Z_{lo}$, where A_1 is $k * L_{hi} + T_{j,hi} + Z_{hi}$, A_2 is $k * L_{lo} + T_{j,lo} + P$ and P is $\log_b(1 + Z/C) - Z$.

18. The article of manufacture of claim 17 wherein if $k * N + j = 0$ for the breakpoint, then $\log_b(X)$ is approximated by $(A_1 + Z_{lo}) + A_2$.

19. The article of manufacture of claim 18 wherein $\log_b(X)$ is otherwise given by $A_1 + (A_2 + Z_{lo})$.

20. The article of manufacture of claim 14 wherein the predetermined value is proportional to $1/(2 * N)$.

21. The article of manufacture of claim 20 wherein $k * L_{hi} + T_{j,hi}$ can be represented without roundoff error for all valid values of k, j .

22. The article of manufacture of claim 21 wherein $T_{0,hi} = T_{0,lo} = 0$ and $T_{N,hi} = L_{hi}$, $T_{N,lo} = L_{lo}$.

23. A computer system comprising:
a processor coupled to a non-volatile storage device, the storage device contains instructions that when executed by the processor approximate a function of a number, by selecting one of a plurality of breakpoints, such that a reduced argument for the function is less than a predetermined value, and evaluating an approximate function of the reduced argument including accessing a look-up table based on the selected breakpoint to obtain a value of a term in the approximate function, wherein the look-up table has at least one breakpoint for which the reduced argument can be computed without roundoff error when the input argument is close to a root of the function.

24. The computer system of claim 23 wherein the function is $\log_b(X)$.

25. The computer system of claim 24 wherein the storage device has further instructions that when executed by the processor represent X in the floating point form $Y \cdot G^k$ where Y is greater than or equal to 1, and wherein the reduced argument is $Z = C \cdot (Y \cdot B_i - 1)$ where C is a function of $\log_b(e)$, and evaluating the approximate function includes determining $\log_b(1/B_i)$ using the look-up table and determining $\log_b(X)$ as an arithmetic combination of at least $k \cdot \log_b(2)$, $\log_b(1/B_i)$, and $\log_b(1+Z/C)$.

26. The computer system of claim 25 wherein $\log_b(1/B_i)$ is given by the look-up table as at least two lower precision values $T_{j,hi}$ and $T_{j,lo}$ whose sum equals $\log_b(1/B_i)$, $\log_b(2)$ is given by at least two lower precision values L_{hi} and L_{lo} whose sum equals $\log_b(2)$, and Z is given by at least two lower precision values Z_{hi} and Z_{lo} whose sum equals Z .

27. The computer system of claim 26 wherein $\log_b(X)$ is approximated by $A_1 + A_2 + Z_{lo}$, where A_1 is $k \cdot L_{hi} + T_{j,hi} + Z_{hi}$, A_2 is $k \cdot L_{lo} + T_{j,lo} + P$ and P is $\log_b(1+Z/C) - Z$.

28. The computer system of claim 23 wherein the processor has a hardware architecture that is deeply pipelined and in which branch mispredictions cause a significant performance penalty.

29. The computer system of claim 28 wherein the processor is one of a plurality of IA-32 series of processors by Intel Corp.